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MODELLING THE CONTACT INTERACTION BETWEEN ROUGH BODIES IN THE PRESENCE OF A LUBRICANT†

Ye. V. KOVALENKO

Moscow

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The axisymmetric problem of the pressure transmission from a circular rigid punch to a linearly deformable foundation through a thin porous elastic layer (coating) adhering to it is studied. The physical and mechanical properties of the coating are described by the equations of Biot's model, while the motion of the viscous compressible fluid inside the pores is governed by Darcy's law of filtration. The problem is reduced to solving an integral equation of the second kind containing Fredholm's operator with respect to the coordinate and Volterra's operator with respect to time. To solve the problem the separation of variables algorithm is used together with asymptotic long and short time methods in the case of a problem with a prescribed domain of contact, and the step-by-step method is used when the boundary of the contact region is not specified. Analytic formulae for the basic characteristics of contact interaction are obtained.

In the domain of contact between the lubricated rough surfaces of interacting bodies the gaps between the microprotuberances are usually filled with the lubricant. When such bodies are pressed together the microprotuberances undergo deformations and the lubricant is compressed, partially escaping from the micropores. Since these processes are localized in thin surface layers of the bodies in contact, it is natural to model them, for example, as elastic or rigid bodies with boundaries reinforced by a think porous elastic coating, the pores of which are filled with the lubricant. Such a coating with its physical and mechanical properties will simulate the processes on a rough surface. In the case of rough surfaces in contact with one another when there is no lubricant the idea of the approach in question was stated by Shtayerman [1]. A slightly different model, taking into account the presence of a lubricant, was proposed in [2, 3].

1. We will first state the basic equations describing the rheological properties of porous elastic media in a system of cylindrical coordinates (r, φ, z) [4]

$$G\Delta\mathbf{u}+(G+\lambda_c)\operatorname{grad}\varepsilon-\alpha M\operatorname{grad}\zeta=0 \tag{1.1}$$

$$\frac{\partial\zeta}{\partial t}=\frac{kM_c}{\eta}\Delta\zeta,\quad M_c=\frac{MG}{\beta(2G+\lambda_c)},\quad \beta=\frac{1-2\nu}{2(1-\nu)} \tag{1.2}$$

$$\tau_{ij}=2G\varepsilon_{ij}+\delta_{ij}(\lambda_c\varepsilon-\alpha M\zeta),\quad \lambda_c=(1-2\beta)G\beta^{-1}+\alpha^2M$$

$$p=-\alpha M\varepsilon+M\zeta,\quad \varepsilon=\operatorname{div}\mathbf{u},\quad \zeta=-\operatorname{div}\mathbf{w}$$

$$\mathbf{w}=f(\mathbf{U}-\mathbf{u}),\quad \mathbf{u}=\{u_r,u_\varphi,u_z\}$$

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Here $\mathbf{u}$ and $\mathbf{U}$ are the displacement vectors of the points of the elastic matrix and the fluid, p is the hydrostatic pressure of the fluid inside the pores τ_{ij} are the components of the stress tensor in the porous medium, ε_{ij} are the components of the strain tensor in the elastic matrix (i and j can be equal to 1, 2, 3, where 1 corresponds to r, 2 to φ, and 3 to z), k is the permeability coefficient of the medium with porosity f, η is the coefficient of viscosity of the fluid component, and G, ν, α and M are the mechanical characteristics of the porous elastic medium, the methods of determining and the physical meaning of which are described in [5]. Moreover, $f \leqslant \alpha \leqslant 1$.

We will consider the auxiliary problem of the action of a normal load $-\sigma(r, \varphi)H(T)$ ($H(t)$ is the Heaviside function) distributed over the domain $0 \leqslant r \leqslant a$, $0 \leqslant \varphi < 2\pi$ on the upper boundary of a thin layer $0 \leqslant z \leqslant h$, $\lambda = ha^{-1} \ll 1$ made from the porous elastic material (1.1), (1.2) and rigidly attached to a non-deformable foundation. We will assume that the surface $z = h$ of the layer is completely permeable, while the foundation is completely impermeable. Then the boundary conditions of the problem have the form

$$z = h: \tau_{zz} = -\sigma(r,\varphi)H(a-r)H(t), \quad \tau_{rz} = \tau_{\varphi z} = 0 \tag{1.3}$$

$$p = \sigma(r,\varphi)H(a-r)H(t)$$

$$z = 0: u_r = u_\varphi = u_z = 0, \quad p_{,z} = 0$$

the layer being free of stresses at infinity.

To state the initial condition we will take into account the presence of air inside the pores (this is not taken into account in Eqs (1.1) and (1.2) of Biot's model). Then at the initial instant the load is applied only to the elastic matrix, while the pores release the air that entered them along with the fluid. This corresponds to the actual conditions of the behaviour of coatings. This fact can be written as follows:

$$t = 0: \qquad p = 0 \quad (\zeta = \alpha\varepsilon) \tag{1.4}$$

Solving the elastic problem for a thin layer of thickness $\lambda \ll 1$ (the method of solution is similar to that presented below) with boundary conditions

$$z = h: \qquad \tau_{zz} = -\sigma(r,\varphi)H(a-r), \quad \tau_{rz} = \tau_{r\varphi} = 0$$

$$z = 0: \qquad u_r = u_\varphi = u_z = 0$$

$$\tau_{zz}, \tau_{rz}, \tau_{r\varphi} \to 0 \quad (r \to \infty)$$

we obtain

$$\varepsilon(r,\varphi,z,0) = -\beta G^{-1}\sigma(r,\varphi) + O(\lambda)$$

Hence, taking (1.4) into account, we find

$$t = 0: \qquad \zeta = -\alpha\beta G^{-1}\sigma(r,\varphi) + O(\lambda) \tag{1.5}$$

To solve the boundary-value problem (1.1)–(1.3), (1.5) we will introduce the dimensionless variables $r = a\rho$, $z = hx$ $(0 \leqslant \rho < \infty,\ 0 \leqslant x \leqslant 1)$ and seek the unknown functions occurring in these equations as the following asymptotic series

$$\{u_r, u_\varphi, u_z, \zeta\} = \sum_{n=0}^{l} \{\Phi_n, \Psi_n, \Gamma_n, \Lambda_n\}\lambda^n + O(\lambda^{l+1}) \tag{1.6}$$

Substituting (1.6) into (1.1)–(1.3), (1.5) and confining ourselves to terms of order zero in the resulting relations, we get

$$\frac{\partial^2\Phi_0}{\partial x^2}=0,\quad \frac{\partial^2\Psi_0}{\partial x^2}=0,\quad \frac{\partial^2\Gamma_0}{\partial x^2}-\frac{\alpha Mh}{2G+\lambda_c}\frac{\partial\Lambda_0}{\partial x}=0 \tag{1.7}$$

$$\frac{\partial^2\Lambda_0}{\partial x^2}-m\frac{\partial\Lambda_0}{\partial t}=0\quad\left(m=\frac{h^2\eta}{kM_c}\right)$$

$$x=0:\quad \Phi_0=\Psi_0=\Gamma_0=\Lambda_{0,x}=0 \tag{1.8}$$

$$x=1:\quad \Phi_{0,x}=\Psi_{0,x}=0,\quad M(-\alpha\Gamma_{0,x}+h\Lambda_0)=h\sigma H(1-\rho)H(t)$$

$$\Gamma_{0,x}=-(1-\alpha)\beta G^{-1}h\sigma H(1-\rho)H(t)$$

$$t=0:\quad \Lambda_0=-\alpha\beta G^{-1}\sigma \tag{1.9}$$

The solution of differential equations (1.7) with boundary conditions (1.8) and initial condition (1.9) can be constructed using a Laplace–Carson transformation in time. Omitting the details, we state the expressions

$$\Phi_0^L=\Psi_0^L=0,\quad \Gamma_0^L=-\frac{\beta h}{G}\left(1-\alpha\frac{\operatorname{th}\sqrt{ms}}{\sqrt{ms}}\right)\sigma \tag{1.10}$$

for the corresponding transforms for $x=1$.

We deduce from (1.6) and (1.10) that a relatively thin porous elastic layer behaves under compression in a similar way as a viscoelastic Fuss–Winkler foundation with an operator bed coefficient, the form of which can be determined by inverting the second formula in (1.10) ($\theta_2(t,\tau)$ is the theta function)

$$u_z(r,\varphi,h,t)=-\frac{\beta h}{G}\left[1-\frac{\alpha}{m}\int_0^t\theta_2\left(0,\frac{t-\tau}{m}\right)d\tau\right]\sigma(r,\varphi)+O(\lambda)\quad(0\leqslant r\leqslant a,t\geqslant 0) \tag{1.11}$$

and in which the displacements u_r and u_φ of the points of the layer are everywhere equal to zero apart from terms $O(\lambda)$.

The solution for an instantaneous distributed load $p_{zz}=-\sigma(r,\ \varphi)\delta(t)$ $(0\leqslant r\leqslant a,\ 0\leqslant\varphi<2\pi)$ acting upon the upper boundary of the layer can be obtained by differentiating (1.11) with respect to t

$$\dot u_z(r,\varphi,h,t)=-\frac{\beta h}{G}\left[\delta(t)-\frac{\alpha}{m}\theta_2\left(0,\frac{t}{m}\right)\right]\sigma(r,\varphi) \tag{1.12}$$

Note that relationships analogous to (1.11) and (1.12) for the case of a plane problem have been found in a more complex way in [6].

2. We will now assume that the thin porous elastic coating is rigidly attached to a linearly deformable foundation [7] with elasticity parameters G_0 and ν_0, the parameter $n=GG_0^{-1}$ being small. In this case, if

$$n=O(\lambda^\gamma)\quad(\lambda\to 0,\gamma>0)$$

then the physical and mechanical properties of the coating can, as before, be modelled by a system of viscoelastic springs [8]. Then the vertical displacement of the points on the upper boundary of the generalized combined linear foundation due to the normal load $-\sigma(r,\ \varphi)\delta(t)$ can be represented as $\dot u_z+\dot\upsilon_z$, where $\dot u_z$ has the form (1.12) and $\dot\upsilon_z$ can be written as follows [7]:

$$\dot{\upsilon}_z(r,\varphi,0,t) = -\frac{\delta(t)}{2\pi\kappa}\int_0^{2\pi}\int_0^a \sigma(\rho,\psi)k\left(\frac{R}{b}\right)d\rho\, d\psi \tag{2.1}$$

$$R=\sqrt{r^2+\rho^2-2r\rho\cos(\varphi-\psi)},\quad 0\leqslant r\leqslant a,\quad 0\leqslant\varphi<2\pi$$

$$k\left(\frac{R}{b}\right)=\frac{1}{b}\int_0^\infty uK(u)J_0\left(u\frac{R}{b}\right)du,\quad \kappa=\frac{G_0}{1-\nu_0}$$

In (2.1) b is the characteristic parameter of the linearly-deformable foundation and $K(u)$ is its kernel, the specific form of which is presented in [7]. Below we consider the case when $\Lambda = ba^{-1} \gg \lambda$ and

$$\begin{aligned} &K(u)>0 \qquad (0\leqslant u<\infty)\\ &K(u)\sim u^{-\gamma_1}(u\to\infty,\gamma_1>1/2),\\ &K(u)\sim u^{\gamma_2}(u\to 0,\gamma_2\geqslant -1)\end{aligned} \tag{2.2}$$

Besides, because the relative thickness of the coating is small, here and henceforth we assume that all boundary conditions can be carried over to the surface $z=0$ of the linearly deformable support.

Proceeding to the study of the corresponding axisymmetric contact problem (Fig. 1) and evaluating the outer integral in the first formula (2.1), we arrive at the following integral equation for the unknown contact pressure $\tau_{rr} = -\sigma(r, t)$

$$\frac{\beta h}{G}\left[\sigma(r,t)-\frac{\alpha}{m}\int_{t_*}^{t}\sigma(r,\tau)\theta_2\left(0,\frac{t-\tau}{m}\right)d\tau\right]+\frac{1}{\kappa}\int_0^a\sigma(\rho,t)\rho k\left(\frac{\rho}{b},\frac{r}{b}\right)d\rho=\gamma(t)-g(r) \tag{2.3}$$

$$0\leqslant r\leqslant a,\quad 0\leqslant t_*\leqslant t\leqslant T<\infty$$

$$k\left(\frac{\rho}{b},\frac{r}{b}\right)=\frac{1}{b}\int_0^\infty uK(u)J_0\left(u\frac{\rho}{b}\right)J_0\left(u\frac{r}{b}\right)du \tag{2.4}$$

where t_* is the time the boundary $r=a$ of the contact domain is reached for an arbitrary point $r_*=a(t_*)$, with $\sigma(r_*, t)$ for $t\leqslant t_*$ when $\dot{a}(t)>0$, and $t_*\equiv 0$ if $\dot{a}(t)\equiv 0$. The case $\dot{a}(t)<0$ will be eliminated from consideration.

To complete the formulation of the problem under investigation we must supplement (2.3) and (2.4) with the quasistatic condition

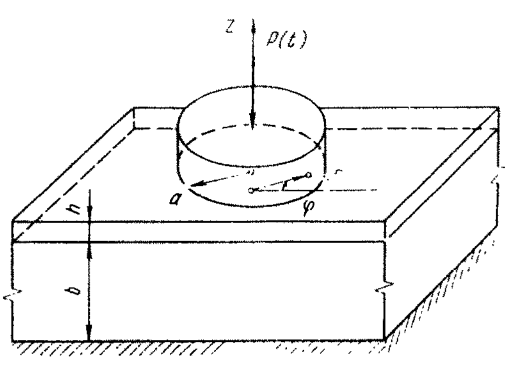

Fig. 1.

$$P(t)=2\pi\int_0^a \rho\sigma(\rho,t)d\rho \tag{2.5}$$

and the relation

$$\sigma(r,t)=0 \quad (r\geqslant a(t)) \tag{2.6}$$

used to find the unknown domain of contact between the punch and the combined linearly deformable foundation for $\dot{a}(t)>0$.

Note that for specified shape $g(r)$ of the punch base, four basic versions of problem (2.3)–(2.6) may be encountered in practice [9]: (1) the functions $\gamma(t)$ and $a(t)=a=\text{const}$ are specified, while $\sigma(r,t)$ and $P(t)$ are to be found; (2) the functions $P(t)$ and $a(t)=a=\text{const}$ are given, while $\sigma(r,t)$ and $\gamma(t)$ are to be found; (3) the rigid displacement $\gamma(t)$ of the punch is specified, and $\sigma(r,t), P(t)$ and $a(t)$ are to be determined; (4) the force $P(t)$ is specified, and $\sigma(r,t), \gamma(t)$ and $a(t)$ are to be found.

3. Consider the case of a contact domain that is constant in time. In the integral equation (2.3) and condition (2.5) we change to the dimensionless variables

$$r=ar',\quad \rho=a\rho',\quad t=t'm,\quad \tau=\tau'm \tag{3.1}$$

and introduce the notation

$$q(r',t')=\sigma(r,t)\kappa^{-1},\quad N(t')=P(t)(a^2\kappa)^{-1}$$

$$\gamma'(t')=\gamma(t)a^{-1},\quad g'(r')=g(r)a^{-1},\quad \mu=\beta\lambda[n(1-\nu_0)]^{-1} \tag{3.2}$$

(below we omit the prime). We get (**I** is the identity operator)

$$\mu(\mathbf{I}-\mathbf{V}_0^t)q+\mathbf{F}_0^1 q=f \quad (0\leqslant r\leqslant 1, 0\leqslant t\leqslant T<\infty) \tag{3.3}$$

$$N(t)=2\pi\int_0^1 q(\rho,t)\rho\, d\rho \tag{3.4}$$

$$\mathbf{V}_s^t\varphi=\alpha\int_s^t \varphi(\tau)\theta_2(0,t-\tau)d\tau,\quad \mathbf{F}_0^a\varphi=\frac{1}{\Lambda}\int_0^a \varphi(\rho)\rho k\left(\frac{\rho}{\Lambda},\frac{r}{\Lambda}\right)d\rho \tag{3.5}$$

Let $f(r,t)=\gamma(t)-g(r)$ be a given continuous function, where $\gamma(t)\in C(0,T)$ is the original function [10] and $g(r)\in C(\Omega)$, where Ω is the circle of a unit radius. By the method of [11] we construct a system of eigenfunctions $\{\varphi_i(r)\}$ $(i\geqslant 1)$ and the corresponding sequence of eigenvalues $\{\alpha_i\}$ of the operator $\mathbf{F}_0^1$ of the form (3.5). By (2.2) and (2.4) this system is orthonormal and complete in the space $L_2(\Omega)$ of square integrable functions. Moreover, $\alpha_i\geqslant 0$ for all i and $\alpha_i\to 0$ $(i\to\infty)$. We shall seek a solution of the integral equation (3.3) in the form [9]

$$q(r,t)=\sum_{i=1}^{\infty}\psi_i(t)\varphi_i(r) \tag{3.6}$$

Substituting (3.6) into (3.3), after obvious algebra we write

$$\left(\mathbf{I}-\frac{\mu}{\mu+\alpha_i}\mathbf{V}_0^t\right)\psi_i=f_i \quad (0\leqslant t\leqslant T) \tag{3.7}$$

$$f_i(t)=\frac{e_i\gamma(t)-g_i}{\mu+\alpha_i},\quad e_i=\int_0^1 r\varphi_i(r)dr,\quad g_i=\int_0^1 g(r)r\varphi_i(r)dr$$

Applying a Laplace–Carson integral transformation in time to Eq. (3.7) and using the convolution theorem [10], we find that

$$\psi_i(t) = f_i(t) + \lambda_i \int_0^t f_i(\tau)\left[\frac{1}{\sqrt{\pi(t-\tau)}} + \lambda_i \exp[\lambda_i^2(t-\tau)]\operatorname{erfc}(-\lambda_i\sqrt{t-\tau})\right]d\tau\,(0 \leqslant t < T^*) \quad (3.8)$$

$$\psi_i(t) = f_i(t) + 2\lambda_i \int_0^t f_i(\tau)\exp\left[-\left(\frac{\pi^2}{4} - 2\lambda_i\right)(t-\tau)\right]d\tau \quad (3.9)$$

$$(T_* < t \leqslant T, \quad \lambda_i = \mu\alpha(\mu + \alpha_i)^{-1})$$

It has been shown in [6] that (3.8) and (3.9) are well matched with one another for $t \in (T_*, T^*)$. Because of this, together they furnish a solution of the integral equation (3.7) in the whole domain of t.

Next, substituting (3.6) into (3.4), we determine the force

$$N(t) = 2\pi \sum_{i=1}^{\infty} e_i \psi_i(t) \quad (3.10)$$

applied to the punch, which produces the required foundation settling $\gamma(t)$. Finally, we construct the solution of the problem in accordance with (3.6) and (3.8)–(3.10).

If the force $N(t)$ applied to the punch is specified ($N(t) \in C(0, T)$ is the original function), the solution of the problem can be constructed using the known scheme [9, 12].

In the case when the contact domain is specified, a mathematical justification of the solutions found in the present section can be obtained by the scheme presented in [11].

4. We will now consider the case of a monotonically increasing area of contact $a(t)$. In (2.3)–(2.6) we change to the dimensionless variables (3.1) and the notation (3.2) with a taken to be $a_0 = a(0)$. Then Eq. (3.3) and formulae (2.6) and (3.4) can be transformed as follows:

$$\mu(\mathbf{I} - \mathbf{V}_{t_*}^t)q + \mathbf{F}_0^a q = f \quad (0 \leqslant r \leqslant a(t), 0 \leqslant t_* \leqslant t \leqslant T < \infty) \quad (4.1)$$

$$q(r,t) = 0 \quad (r \geqslant a(t)) \quad (4.2)$$

$$N(t) = 2\pi \int_0^{a(t)} q(\rho, t)\rho\, d\rho \quad (4.3)$$

where the prime is again omitted in the dimensionless variables $a'(t') = a(t)a_0^{-1}$ and $t'_* = t_* m^{-1}$.

We observe that it is very difficult to compute the exact solution of this problem. In what follows we shall therefore confine ourselves to constructing a simpler approximate solution. We will use an analogue of the step-by-step method [13].

We divide the time interval [0, T] into M fairly small intervals, in each of which we approximate the contact pressure $q(r, t)$ and the radius $a(t)$ of the contact domain by functions that are piecewise-constant in time

$$t_{m-1} \leqslant t < t_m: \quad q(r,t) = q(r, t_{m-1}), a(t) = a(t_{m-1}) = a_{m-1}$$
$$m = 1, 2, \ldots, M; \quad t_0 = 0; \quad t_M = T; \quad a_0 = 1$$

As a result, in place of (4.1) we arrive at a sequence of elasticity problems with increasing areas of contact

$$\mu q_0 + \mathbf{F}_0^1 q_0 = \gamma_0 - q \quad (q_0(r) = q(r, 0), \quad \gamma_0 = \gamma(0), \quad 0 \leqslant r \leqslant 1) \quad (4.4)$$

$$\mu q(r,t_m)+\mathbf{F}_0^{a_m}q(\rho,t_m)=f_m(r,t_m)\quad(0\leqslant r\leqslant a_m)\tag{4.5}$$

$$f_m(r,t)=\gamma(t)+\mu\sum_{i=0}^{m-1}\mathbf{V}_{t_i}^{t_{i+1}}q(r,t_i)-g(r)\qquad m=1,2,\dots,M-1$$

which increase because new rough microscopic areas filled with the lubricant are being successively involved in the process. In other words, the contact pressure and the radii of the new areas of contact are successively computed using the solutions of (4.4) and (4.5) satisfying conditions (4.2) and (4.3)

$$q(r,t_m)=0\qquad(r\geqslant a_m)\tag{4.6}$$

$$N(t_m)=2\pi\int_0^{a_m}q(\rho,t_m)\rho\,d\rho\quad(m=0,1,\dots,M-1)\tag{4.7}$$

and obtained at the previous time steps.

Let $0\leqslant t<t_1$. We transform Eq. (4.4) as follows. To begin with we set $r=1$ in it and use condition (4.6). Then we multiply both sides of (4.4) by $2\pi r\,dr$, integrate from 0 to 1, and take (4.7) into account with $m=0$ to get

$$\int_0^1 q_0(\rho)\rho k\left(\frac{\rho}{\Lambda},\frac{1}{\Lambda}\right)d\rho=\gamma_0-g(1)\tag{4.8}$$

$$\mu N(0)+2\pi\int_0^1 r\mathbf{F}_0^1 q_0(\rho)dr=\pi\left[\gamma_0-2\int_0^1 rg(r)dr\right]$$

We simplify (4.8) by replacing $q_0(r)$ by its mean value $N(0)\pi^{-1}$. On taking quadratures, we use (2.4) to write

$$N(0)\int_0^\infty K(u)J_0\left(\frac{u}{\Lambda}\right)J_1\left(\frac{u}{\Lambda}\right)du=\pi[\gamma_0-g(1)]\tag{4.9}$$

$$N(0)\left[\mu+2\Lambda\int_0^\infty K(u)u^{-1}J_1^2\left(\frac{u}{\Lambda}\right)du\right]=\pi\left[\gamma_0-2\int_0^1 rg(r)dr\right]$$

Relationships (4.9) can be used to find Λ and $N(0)$ with specified settling γ_0, or to find γ_0 when $N(0)$ is specified. We remark that an efficient method of computing the improper integrals of rapidly oscillating functions occurring in (4.9) is presented in [14].

After determining the necessary parameters from system (4.9) and substituting them into Eq. (4.4), we obtain a solution of the latter using, for example, the eigenfunctions of the integral operator $\mathbf{F}_0^1$ (see Section 3). The solution constructed can be checked by verifying conditions (4.6) and (4.7) to within the accuracy specified in practice. When the conditions are satisfied, we proceed to the next interval, and so on. As a result, at the mth step we have Eq. (4.5), the solution of which must satisfy the conditions (the analogues of (4.9) for $m\geqslant1$)

$$N(t_m)\int_0^\infty K(u)J_0\left(\frac{u}{\Lambda}a_m\right)J_1\left(\frac{u}{\Lambda}a_m\right)du=\pi a_m[\gamma(t_m)-g(a_m)]$$

$$N(t_m)\left[\mu+2\Lambda\int_0^\infty K(u)u^{-1}J_1^2\left(\frac{u}{\Lambda}a_m\right)du\right]=$$

$$=\pi\left[a_m^2\gamma(t_m)+\frac{\mu}{\pi}\sum_{i=0}^{m-1}N(t_i)\int_{t_i}^{t_{i+1}}\theta_2(0,t_m-\tau)d\tau-2\int_0^{a_m}rg(r)dr\right]$$

As above, such a problem can be solved using the eigenfunctions of $\mathbf{F}_0^{a_m}$, which can be

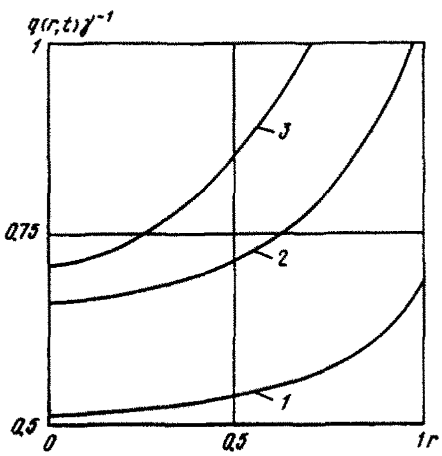

Fig. 2.

constructed by the method of [11] taking the sequence of Legendre polynomials $P_i[1-2(ra_m^{-1})^2]$ as the basis.

5. As an example, we present a numerical computation of the mechanical characteristics of the contact problem of indenting a punch of circular cross-section and a flat base into an elastic layer (Fig. 1), the lower side of which is rigidly fixed and the upper side of which is reinforced by a thin porous elastic coating. We will assume that the contact area does not change and the prescribed punch settling is constant in time. In the dimensionless variables (3.1) and taking the notation (3.2) into account, this problem reduces to the integral equation (3.3), where

$$f(r,t)=\gamma=\text{const}$$
$$K(u)=\frac{2\kappa_0\,\text{sh}\,2u-4u}{u(2\kappa_0\,\text{ch}\,2u+4u^2+1+\kappa_0^2)}\qquad(\kappa_0=3-4\nu_0)$$

Next, we set $\nu_0=0.3$, $\Lambda=6$, and $\mu=1$, and we vary the parameter α, which characterizes the change of volume of the porous medium relative to the dilation of the elastic matrix for $p=0$ (the case $\alpha=1$ corresponds to an incompressible fluid).

Graphs of the distribution of the contact pressure $q(r,t)\gamma^{-1}$ are shown in Fig. 2: curve 1 corresponds to $t=0$ (dry contact), curve 2 to $t=\infty$ and $\alpha=0.5$, and curve 3 to $t=\infty$ and $\alpha=0.99$. The values of $N(t)\gamma^{-1}$ computed from (3.4) are equal to 1.82; 2.57; 4.55. It can be seen that the presence of the lubricant in the contact domain as well as its properties, in particular compressibility, have a considerable effect on the properties of the thin surface layer, as if they were strengthening the layer in the same way as surface strengthening by various techniques.

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